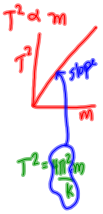
Chapter 13 - Simple Harmonic Motion

Mass-Spring System:

$T = 2\pi\sqrt{\frac{m}{k}}$ → $T \propto \sqrt{m}$

$T^2 \propto m$



where T is the period (s)
m is the mass (kg)
k is the spring constant ($\frac{N}{m}$)

Also look at Conservation of Energy

$E_k = \frac{1}{2}mv^2$

$E_e = \frac{1}{2}kx^2$

$E_{final} = E'_{final}$

Hooke's Law → $F_e = kx$

Example

$x = 12.0\,cm$

$m = 125\,g$

20 cycles in 15.5 s

a) $T = ?$

*b) $k = ?$

c) $E_{final} = ?$

d) $V_{max} = ?$

e) $V = ?$, $x = 10.0\,cm$

a) $T = \frac{15.5\,s}{20\,cycles} = 0.775\,s$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = \frac{4\pi^2 m}{k}$

$k = \frac{4\pi^2 m}{T^2}$

$k = \frac{4\pi^2(0.125\,kg)}{(0.775\,s)^2}$

$k = 8.22\,\frac{N}{m}$

c) At maximum displacement (i.e. 12.0 cm)

$E_{final} = \cancel{E_k}^{0} + E_e$

$E_{final} = \frac{1}{2}kx^2$

$E_{final} = \frac{1}{2}\left(8.22\,\frac{N}{m}\right)(0.120\,m)^2$

$E_{final} = 0.0592\,J$

d) $V_{mass} = ?$ (V_{max} occurs when the mass is passing through equilibrium)

($E_e = 0$ here)

All the energy is E_k!!

$E_k = \frac{1}{2}mv^2$

$0.0592\,J = \frac{1}{2}(0.125\,kg)v^2$

$v^2 = \frac{2(0.0592\,J)}{0.125\,kg}$

$v = \pm 0.973\,m/s$

e) $V = ?$ when $x = 0.100\,m$?

$E_{final} = E_k + E_e$

$0.0592\,J = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$0.0592\,J = \frac{1}{2}(0.125\,kg)v^2 + \frac{1}{2}\left(8.22\,\frac{N}{m}\right)(0.100\,m)^2$

$0.0592\,J = \frac{1}{2}(0.125\,kg)v^2 + 0.0411\,J$

(E_{final}, E_k, E_e)

$0.0181\,J = \frac{1}{2}(0.125\,kg)v^2$

$v^2 = 0.289\,\frac{m^2}{s^2}$

$v = \pm 0.538\,m/s$

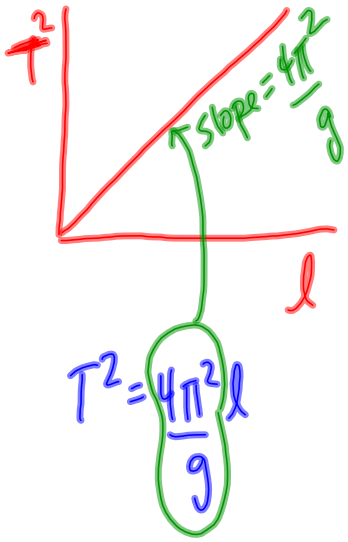
\+ stretches
\- compress.

Pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

$T \propto \sqrt{l}$

$T^2 \propto l$

where T is the period (s)
l is the length (m)
g is $9.81\ m/s^2$



$T^2 = \frac{4\pi^2}{g} l$

Consider the conservation of Energy:

$E_g = mgh$

$E_{total} = E'_{total}$

$E_g + E_k = E'_g + E'_k$

E_k is max at bottom of swing ($\therefore$ v is max)

To Do

- PP|608
- look over MP|613
- PP|614